Section 4.6

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In Exercises 76-78, find the domain, asymptotes and holes of the rational function.

76

(76) $f(x) = \dfrac{3x - 5}{2x + 1}$

The denominator is zero when $2x + 1 = 0$ or $x = \frac{-1}{2}$.
Therefore, the domain is $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{-1}{2}, \infty\right)$

Since the degree of the numerator and denominator are equal (both equal to 1) and the ratio of their leading coefficients is 3/2, then the horizontal asymptote is $y = 3/2$.

Since the denominator is zero when $x = \frac{-1}{2}$ and it is not removable, then
the vertical asymptote is $x = \frac{-1}{2}$

There are no values of "x" that will make the numerator and denominator both equal to zero so there are no holes.

Domain: $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{-1}{2}, \infty\right)$
Horizontal Asymptote: $y = 3/2$
Vertical Asymptote: $x = -1/2$
Holes: none

77

(77) $f(x) = \dfrac{x^2 + 4x - 5}{x^2 + 8x + 15}$

The denominator is zero when
$x^2 + 8x + 15 = 0$
$(x + 3)(x + 5) = 0$
$x + 3 = 0$ or $x + 5 = 0$
$x = -3$ or $x = -5$

Therefore, the domain is $(-\infty, -3) \cup (-3, -5) \cup (-5, \infty)$

Since the degree of the numerator and denominator are equal (both equal to 2) and the ratio of their leading coefficients is 1/1 = 1, then the horizontal asymptote is $y = 1$.

The factored form of the function is $f(x) = \frac{(x-1)(x+5)}{(x+3)(x+5)}$
Since the denominator is zero when $x = -3$ and it is not removable, then
the vertical asymptote is $x = -3$
The value of "x" that will make the numerator and denominator both equal to zero is -5
The simplified form of f is $g(x) = \frac{x-1}{x+3}$ and $g(-5) = \frac{-5-1}{-5+3} = \frac{-6}{-2} = 3.$
Therefore, the hole is at $(-5,3)$

Domain: $(-\infty, -3) \cup (-3, -5) \cup (-5, \infty)$
Horizontal Asymptote: $y = 1$
Vertical Asymptote: $x = -3$
Hole: $(-5,3)$

#78

(78) $f(x) = \frac{x-5}{x^2-2x-3}$

The denominator is zero when
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x + 1 = 0$ or $x - 3 = 0$
$x = -1$ or $x = 3$
Therefore, the domain is $(-\infty, -1) \cup (-1,3) \cup (3, \infty)$

Since the degree of the numerator is less than the degree of the denominator (1 < 2), then the horizontal asymptote is $y = 0$.

Since the denominator is zero when $x = -1$ and $x = 3$ and they are not removable, then
the vertical asymptotes are $x = -1$ and $x = 3$

There are no values of "x" that will make the numerator and denominator both equal to zero so there are no holes.

Domain: $(-\infty, -1) \cup (-1,3) \cup (3, \infty)$
Horizontal Asymptote: $y = 0$
Vertical Asymptote: $x = -1$ and $x = 3$
Holes: none

79

(79) Solve: $\frac{3x+1}{x-2} = 0$

Since the fraction is zero, then the numerator must be zero.
$3x + 1 = 0$
$x = \frac{-1}{3}$

80

(80) Solve: $\frac{2x+3}{x-1} = 3$

Multiply both sides by $x - 1$
$2x + 3 = 3x - 3$
Subtract $2x$ from each side
$3 = x - 3$
Add 3 to both sides
$6 = x$
$x = 6$

In Exercises 81-83, sketch the graph of $y = f(x)$ using the 8-step procedure in this section (see notecard).

81

(81) $f(x) = \frac{x-1}{x-2}$

1) Vertical Asymptote: $x = 2$
2) Horizontal Asymptote: $y = 1$ because the numerator and denominator have the same degree and the ratio of their leading coefficients is 1/1 = 1
3) $f(0) = 1/2$ so the y-intercept is (0, 1/2)
4) $f(x) = 0$ when x= 1 so the x-intercept is (1, 0)
5) $f(x) = 1$ has no solution so the graph does not intersect the horizontal asymptote
   Calculation:
   $\frac{x-1}{x-2} = 1$
   Multiply both sides by $x - 2$

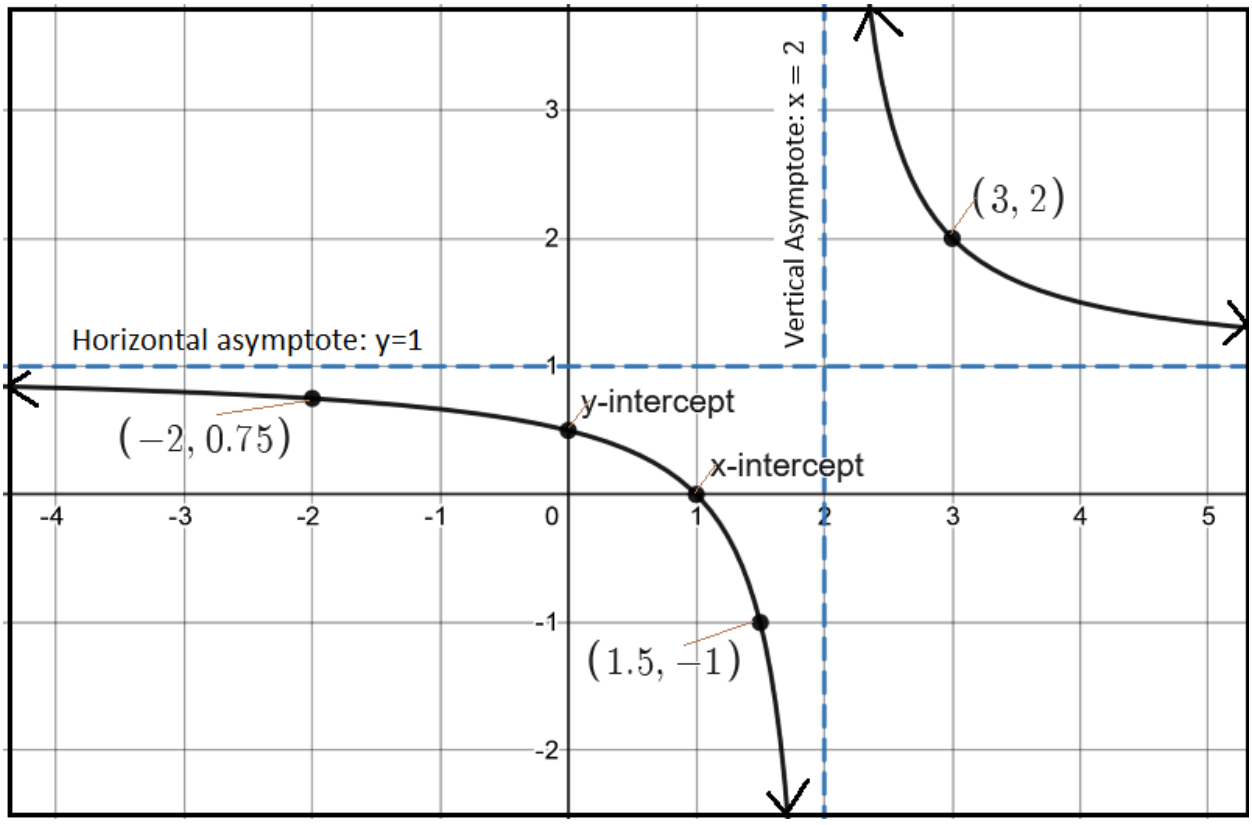
$x - 1 = x - 2$
Subtract x from each side
$-1 = -2$ which shows $f(x) = 1$ has no solution

6) Table of Values

x	-2	1.5	3
f(x) =(x-1)/(x-2)	0.75	-1	2

Therefore, the graph contains the points (-2, 0.75), (1.5,-1), and (3,2)

7) Graph:



8) There are no holes because there are no values of "x" that will make the numerator and denominator both equal to zero

82

(82) $f(x) = \frac{x^2 + x - 2}{x^2 - 4}$

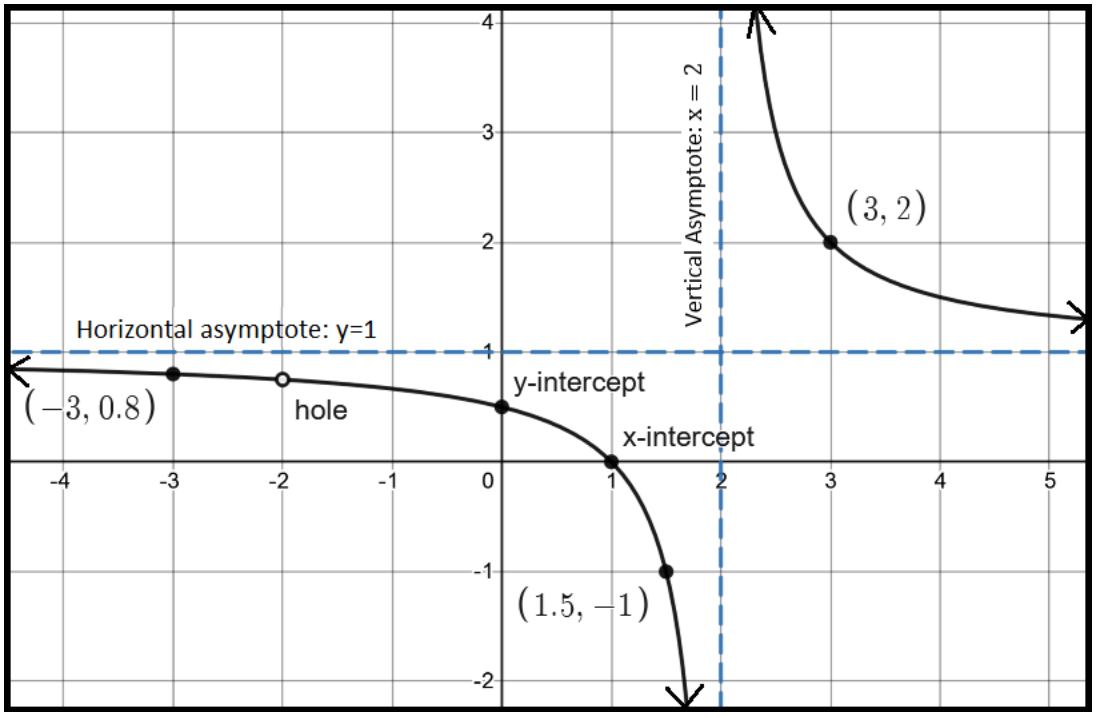
$$f(x) = \frac{(x+2)(x-1)}{(x+2)(x-2)} = \frac{x-1}{x-2}, x \neq -2$$

1) Vertical Asymptote: $x = 2$
2) Horizontal Asymptote: $y = 1$ because the numerator and denominator have the same degree and the ratio of their leading coefficients is 1/1 = 1
3) $f(0) = 1/2$ so the y-intercept is (0, 1/2)
4) $f(x) = 0$ when x= 1 so the x-intercept is (1, 0)
5) $f(x) = 1$ has no solution so the graph does not intersect the horizontal asymptote
   Calculation:
   $\frac{x-1}{x-2} = 1$
   Multiply both sides by $x - 2$
   $x - 1 = x - 2$
   Subtract x from each side
   $-1 = -2$ which shows $f(x) = 1$ has no solution

6) Table of Values

x	-3	1.5	3
f(x) =(x-1)/(x-2), $x \neq -2$	0.8	-1	2

7) Graph:



8) There is a hole at (-2, 0.75)

Note: The graph in # 81 and # 82 are the same except that in # 82 there is a hole at (-2, 0.75)

83

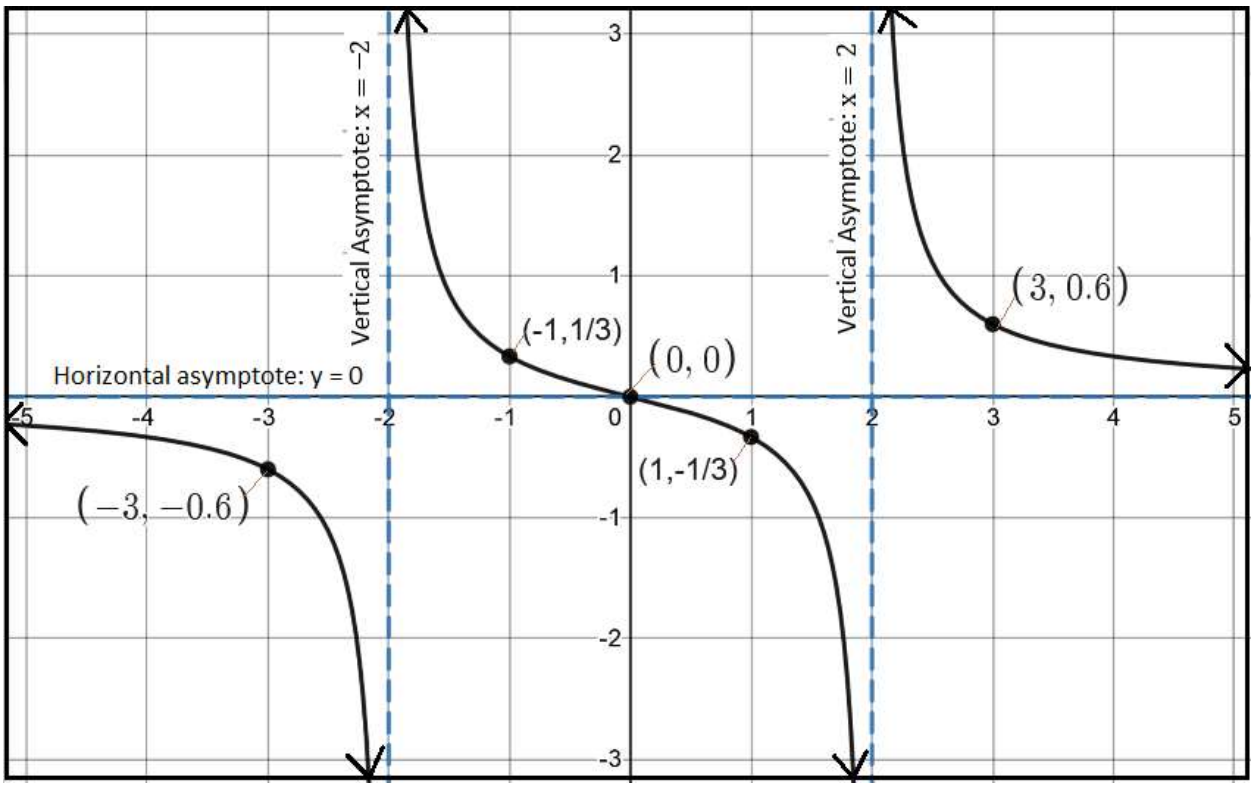
(83) $f(x) = \frac{x}{x^2 - 4}$

$$f(x) = \frac{x}{(x-2)(x+2)}$$

1) Vertical Asymptote: $x = 2$ and $x = -2$
2) Horizontal Asymptote: $y = 0$ because the degree of the numerator is less than the degree of the denominator
3) $f(0) = 0$ so the y-intercept is (0, 0)
4) $f(x) = 0$ when x = 0 so the x-intercept is (0, 0)
5) $f(x) = 0$ when x= 0 so the graph intersects the horizontal asymptote at (0,0)
6) Table of Values

x	-3	-1	1	3
$f(x) = x/(x^2 - 4)$	-0.6	1/3	-1/3	0.6

7) Graph:



8) There are no holes because there are no values of "x" that will make the numerator and denominator both equal to zero.

84

(84) Sketch the graph of Exercise 81 using transformations.
(Hint: $\frac{x-1}{x-2} = \frac{1}{x-2} + 1$.)

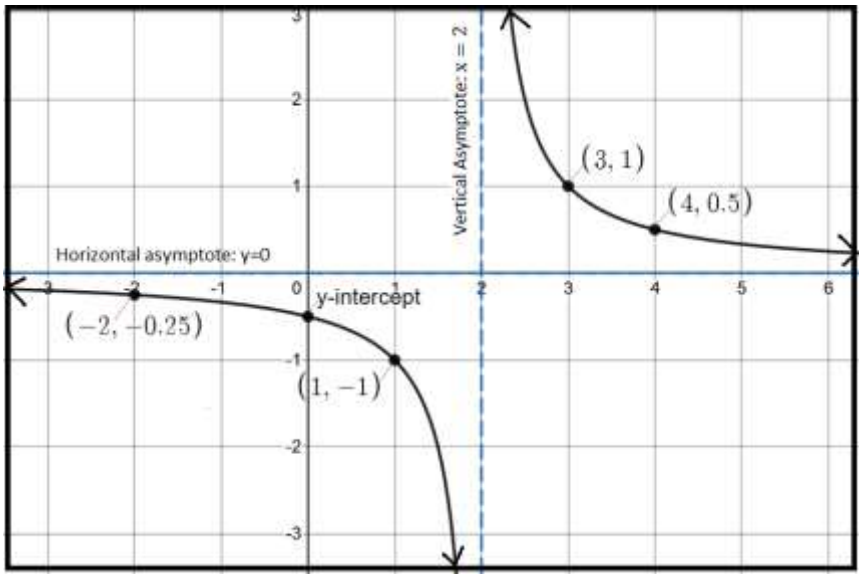
Let $f(x) = \frac{1}{x-2}$

1) Vertical Asymptote: $x = 2$
2) Horizontal Asymptote: $y = 0$ because the degree of the numerator is less than the degree of the denominator
3) $f(0) = -1/2$ so the y-intercept is (0, -1/2)
4) $f(x) = 0$ has no solution because the numerator cannot be equal to zero so there is no x-intercept
5) The graph does not intersect the horizontal asymptote because f(x) = 0 has no solution
6) Table of Values

x	-2	1	3	4
f(x) = 1/(x-2)	-0.25	-1	1	0.5

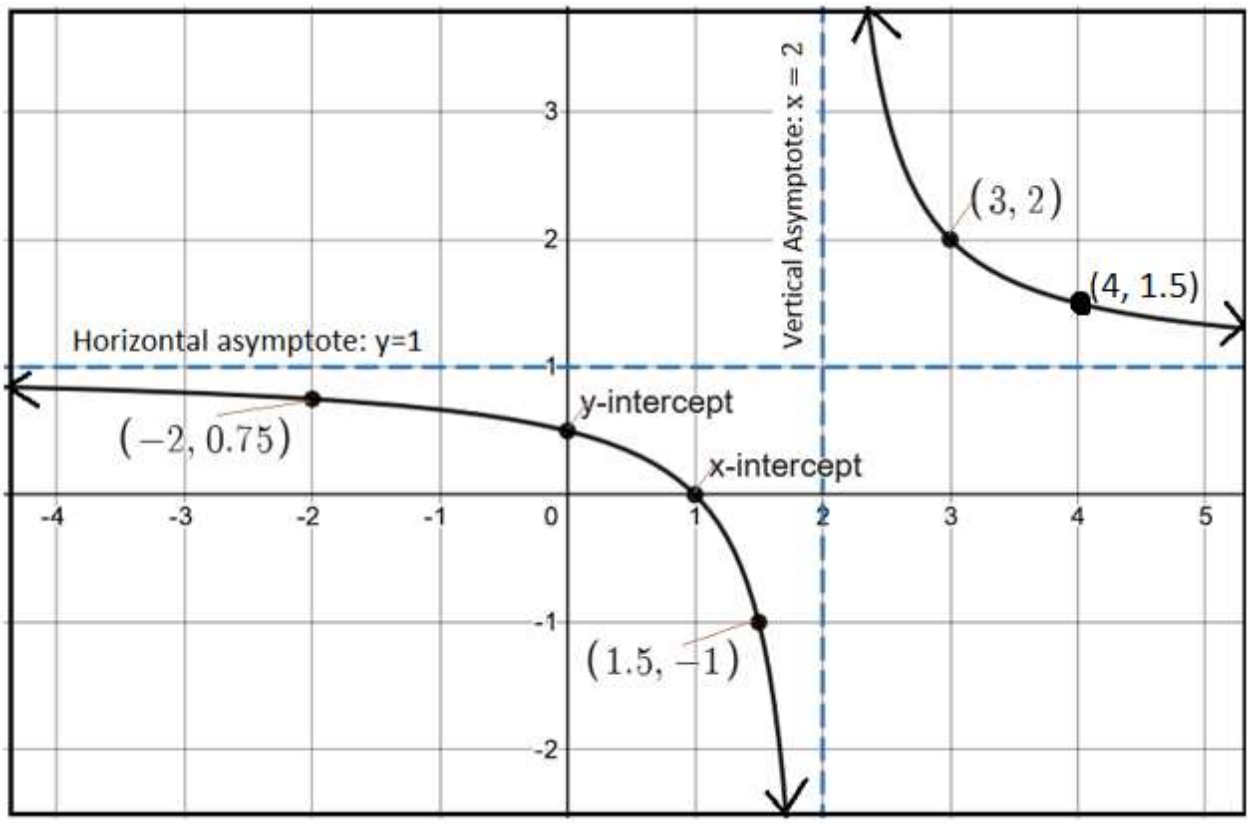
Therefore, the graph contains the points (-2, -0.25), (1,-1), (3,1), and (4, 0.5)

7) Graph of $f(x) = \frac{1}{x-2}$



8) There are no holes because there are no values of "x" that will make the numerator and denominator both equal to zero.

Move the entire graph by one unit upward to graph $y = \frac{1}{x-2} + 1$



85

(85) Find the slant asymptote of $f(x) = \frac{4x^2 - 5x + 1}{x - 2}$.

The degree of the numerator is one more than the degree of the denominator so to get the slant asymptote, we divide the numerator by denominator using synthetic division.

2	4	-5	1
		8	6
	4	3	7

Therefore, $f(x) = 4x + 3 + \frac{7}{x-2}$ and the slant asymptote is $y = 4x + 3$